

## CHAPTER 1

### A FORMAL LANGUAGE WITH NON-DISTRIBUTIVE PLURALS: PRELIMINARY CONSIDERATIONS

#### Non-distributive predication

Standard first-order logic does not provide adequate resources for properly representing many ordinary things that we say.

(1) Arnie, Bob and Carlos are shipmates.<sup>1</sup>

This is something true of the three of them together. We cannot say

Arnie is a shipmate

except perhaps as elliptical for something that connects Arnie to others. (Arnie is a shipmate *of someone*.) Many predicates can be true of some things without being true of any one of them. For example:

They are shipmates (classmates, fraternity brothers)

They are meeting together

They lifted a piano

They are surrounding a building

They come from many different countries

They weigh over 500 pounds

Standard systems of logic provide no place for such predication.

Let's say that a predicate F is *distributive*<sup>2</sup> if the following condition holds in virtue of the meaning of the predicate F:

Whenever some things are F, each one of them is F.

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<sup>1</sup> The predicate 'are shipmates' is the principal example in Massey 1976.

<sup>2</sup> Unfortunately, the word 'distributive' has been used for a wide variety of different things in discussions in logic and metaphysics. In using it here, though, I am following an established usage, not introducing a new one.

For example:

Whenever some things are baboons, each one of them is a baboon.

In standard first-order logic, every predicate is distributive. The concept of a single individual satisfying a predicate is fundamental; saying that some individuals satisfy a predicate can only mean that each of them satisfies it. Ordinary quantification is built on this. The formulas:

$$\exists x Fx$$

$$\forall x Fx$$

are true just when at least one individual (each individual) satisfies 'Fx'. We can always look at individuals in the domain separately in deciding what satisfies the predicate.

Although restricted quantification has many advantages over standard unrestricted quantification,<sup>3</sup> it has usually been limited in the same way to distributive predication.

$$[\exists x: Fx] Gx \quad (\text{Some F are G})$$

$$[\forall x: Fx] Gx \quad (\text{Every F is G})$$

These are true when at least one (each) thing that satisfies 'Fx' also satisfies 'Gx'. We look at each individual separately to determine whether the predicates are satisfied. There is no provision for non-distributive predication.

As our examples illustrate, though, many ordinary predicates are not distributive. When some people surround a building, they satisfy the predicate '... are surrounding the building' together, not separately. The principal goal of this work is to explore the semantics of non-distributive predication and to look at the consequences of extending ordinary first-

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<sup>3</sup> See Barwise and Cooper 1981, Neale 1990 or Brown 1984 for some accounts of the advantages of restricted quantification. Neale 1990, pp. 38-44, is a very useful brief discussion of restricted quantification. McKay 1989 (revised 2005) presents an elementary introduction to restricted quantification (Chapter 10).

order logic to allow for such predication.<sup>4</sup> We will develop the semantics for a language with restricted quantification. This enables us to do a better job of modeling the features of English sentences that include such predicates. It also allows us to consider a full set of quantifier concepts, including *several*, *many*, *most*, and cardinal quantifiers (*exactly n*, *at least n*, etc.), in addition to universals and existentials.<sup>5</sup> We will look at the way that both conjoined noun phrases and quantifier phrases combine with such predicates to form sentences.

In addition to distributivity, there is another limitation on the predicates of traditional systems of logic. In traditional logic, all predicates are *cumulative*:<sup>6</sup>

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<sup>4</sup> Byeong-Uk Yi's work has developed the semantics for plural predication and quantification within a system of unrestricted quantification. See especially his 2005, "The Logic and Meaning of Plurals." His work is full of valuable insights. My work initially developed independently of his, but it is largely consistent with his approach, though my work develops a language with restricted quantification, and, accordingly, a larger range of quantifiers and a more immediate link with natural language. Hossack 2000 also develops plural quantification within a framework that employs only unrestricted quantification. In linguistics, the work of Scha, Link, Landman, Schwarzschild and Lasersohn (among others) also contains many ideas that are important to developing a semantics for plural predication and quantification.

<sup>5</sup> Agustín Rayo (2002) has also proposed a semantics for generalized quantifiers, but his treatment does not take account of a problem for proportional quantifiers that introduces semantic anomaly. This problem will be discussed in Chapter 3.

<sup>6</sup> Schwarzschild 1996, p. 11 introduces this use of the term *cumulativity*, with some indication of its relation to earlier uses of *cumulative* (in Landman 1989 and Quine 1960) and also to a contrasting use of the term in Scha 1984.

A monadic predicate  $F$  is cumulative iff the meaning of the predicate requires that whenever some things  $X$  are  $F$  and some things  $Y$  are  $F$ , then  $X$  and  $Y$  together are  $F$ .<sup>7</sup>

For example, if Alice, Betty and Carla are students, and Dave, Ernie and Frank are students, then Alice, Betty, Carla, Dave, Ernie and Frank are students. Many (though not all) non-distributive predicates are also non-cumulative. For example, even if Alice, Betty and Carla are classmates, and Dave, Ernie and Frank are classmates, it does not follow that Alice, Betty, Carla, Dave, Ernie and Frank are classmates. And even if some students  $X$  are sitting in a circle and some students  $Y$  are sitting in a circle, it does not follow that  $X$  and  $Y$  together are sitting in a circle.<sup>8</sup> There are also some predicates that are distributive and not cumulative, for example:

They are fewer than four in number.

They are a minority.

They are of just one gender.

They are odd in number.

Extending ordinary first-order logic to allow for such non-distributive and non-cumulative predication will give us resources for representing a wider range of arguments, resources for expressing some philosophical theories in ways that makes their ontological claims clearer, and resources that might be useful in developing natural language semantics.

Because of the relative rarity of non-cumulative predicates that are distributive, I will usually refer to our task as the task of accommodating non-distributive predication.

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<sup>7</sup> For general applicability to infinite domains, we need a slightly different definition: a monadic predicate  $F$  is cumulative iff the meaning of  $F$  requires that for any things  $X$ , if whenever some things  $Y$  are properly among  $X$ ,  $Y$  are  $F$ , then  $X$  are  $F$ .

<sup>8</sup> Tyler Burge 1977 presents a theory of aggregates to deal with non-distributive plural predication, but his account will not give the right results for non-cumulative predicates. (Cf. the "analog to set-abstraction" on p. 100 of Burge 1977.)

That can be taken as shorthand for *non-distributive and non-cumulative* predication.<sup>9</sup> Really what we want is a notion of joint satisfaction of a predicate, contrasted with individual satisfaction.<sup>10</sup>

We will develop this formal language with ordinary language in mind, as a guide to what can be said and so might be incorporated. Some predicates allow for both distributive and non-distributive satisfaction, producing ambiguous sentences. ("They weigh 500 pounds" for example.) We will see that there are two ways to think of this: we can regard the argument position as the locus of the ambiguity, or we can identify an implicit (unexpressed) "distributing universal" in the distributive sentence as the source of the ambiguity. (Linguists have differed about the locus of the ambiguity, about whether it is in predicates or singular terms, and our work provides an alternative approach.) We will identify "distributive correlates" of non-distributive predicates that sometimes mislead us in considering examples. We will find that we need to distinguish between two very different universal quantifiers in connection with quantification with non-distributive predicates. We will also need to make a distinction among quantifiers, between the proportional and the

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<sup>9</sup> For completeness, we also note that there are predicates that are cumulative and not distributive, such as "They are more than four in number," "They are a majority of the students," "They come from more than seven countries," etc..

<sup>10</sup> One might also define a concept of strong distributivity: whenever some things  $X$  are  $F$ , if  $Y$  are among  $X$ , then  $Y$  are  $F$ . The predicate ' $X$  are odd in number' is distributive but not strongly distributive, given our definitions. Such predicates are in any case non-cumulative, so the differentiation is of little significance.

Another definition of distribution is also current. Let's say that a predicate  $F$  is distributive\* if and only if the following holds: some things are  $F$  if and only if each of them is  $F$ . (This is an adaptation of Oliver and Smiley's definition of distribution.) A predicate is distributive\* if and only if it is distributive and cumulative. My separation of the two highlights cumulativity, a property of some independent interest.

non-proportional, and we will recognize a source of semantic anomaly in the use of proportional quantifiers that has not been widely recognized before.<sup>11</sup> In providing a

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<sup>11</sup> There are many things that this work will not accomplish. The intent is to develop a formal language with non-distributive predication and quantification. We will not deal with every issue of interest concerning plurals in natural language. For example, David Dowty has considered some similar issues concerning non-distributive predication, but his principal goal was to understand the varying uses of *all*. I will present some semantic considerations that may help with the foundations of that particular project. In particular, the recognition of two distinct universals, the systematic treatment of the difference between distributive and non-distributive argument places, and the explicit representation of distributing universals provide us with resources for the analysis of natural language quantifiers. I will not, however, attempt to account for the full variety of uses of *all* in English in this discussion. Another example: I will provide a way of representing so-called "cumulative readings" of multiply quantified sentences (in Chapter 4), but I will not answer the (perhaps pragmatic) question of how, precisely, the use of quantifier expressions in English can produce such an interpretation as the principal interpretation of a sentence. Also, I make the usual logicians' assumption that a common noun like 'student' in 'All students' is to be represented in terms of the predicate 'x is a student'. There are reasons to treat the common noun as something that differs from a true predicate and to see such predicates as requiring analysis in terms of the common noun, rather than vice versa. That is an issue that is, I think, independent of any issue involving plurals, and I continue with the standard logicians' assumption for simplicity in exposition.

My goals may contrast with the goals of many linguists. I plan to enrich predicate logic with new expressive resources, and I will look at many interesting examples from ordinary language to illustrate the use of these new resources. The linguist must say in general how what can be said with such resources is said in natural language, and give a systematic theory of the connection. My illustrations provide suggestions, but not a full linguistic theory.

semantics and avoiding "singularism" (see Chapter 2), we will make it clear that using plural pronouns and quantifiers and non-distributive predicates in our semantics can expand the expressive scope of our formal language in an important way.

### **Distributive and non-distributive plurals: some basics**

Many plural sentences, those involving only distributive, cumulative predicates, have seemed unproblematic from the standpoint of standard systems of logic.

- (2) Arnie, Bob and Carlos are students (read, live in New York City).
- (3) (a) Some students read.  
(b) All students read.

Within the logical tradition, we have generally represented sentences like (2) as conjunctions, viewing (2) as an ellipsis for:

- (2') Arnie is a student, Bob is a student, and Carlos is a student.

Since the predicate 'are students' is distributive, (2) and (2') must be equivalent. No such ellipsis is at work in (1), however.

- (1\*) \*Arnie is a shipmate, Bob is a shipmate, and Carlos is a shipmate.

Similarly, the representation of (3) in ordinary first-order logic (with restricted quantifiers) is the following:<sup>12</sup>

- (4) (a)  $[\exists x: Sx] Rx$  (Some  $x$  such that  $x$  is a student satisfies this condition,  $x$  reads)

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<sup>12</sup> This example could also be represented with unrestricted quantification.

- (3U)  $\exists x (Sx \wedge Rx)$  (Some  $x$  is such that  $x$  is a student and  $x$  reads books)

- $\forall x (Sx \supset Rx)$  (Every  $x$  is such that if  $x$  is a student then  $x$  reads books)

We employ restricted quantification because it provides the resources for a fuller set of quantifiers (*most* and *many*, for example, in addition to the universal and existential).

- (b)  $[\forall x: Sx] Rx$  (Every  $x$  such that  $x$  is a student satisfies this condition,  $x$  reads)

Although these are singular and the English is plural, these seem to capture adequately the sense of the English. Clearly, though, no such logical formulas can capture the sense of sentences involving non-distributive predicates.

- (5) Some students surrounded Adams Hall.  
(6) All students in my class are shipmates.

No single student satisfies the predicate ' $x$  surrounded the building' (at least in the typical situation) or ' $x$  is a shipmate', and so nothing like (4) will work in representing these sentences.

Sentences like (5) and (6) have another feature of importance. They involve connecting a distributive predication ("are students") with a non-distributive predication ("are surrounding Adams Hall" and "are shipmates"). It seems that a single (plural) variable can have both distributive and non-distributive occurrences within a sentence. Our ultimate theory will need to reflect this in some way.

Our goal might be described as the development of a formal language that can be a framework for the general understanding of plurals, of plural quantification, of non-distributive and non-cumulative predications, and of non-distributive or plural argument positions in predicates and relations. We need to fit all of these together.

### **Conjoined predicates and argument-place distinctions**

The problem of providing a place for non-distributive plurals also calls into question the treatment of other plurals as conjunctions. If we regard (2) as fundamentally a conjunction, then we treat the subject terms in (1) and (2) in different ways.<sup>13</sup> However, we

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<sup>13</sup> In what follows we are arguing against Frege (among others), who favored this disparity of treatment. Frege 1914, pp. 227-228:

can conjoin the predicates in (1) and (2) to produce a sentence in which they have the same plural term as subject:

(7) Arnie, Bob and Carlos are students and are shipmates.

(8) Arnie, Bob and Carlos are shipmates who are students,

So it appears that we must after all have a unified account of plural terms that allows for distributive and non-distributive satisfaction of predicates.<sup>14</sup> That suggests that we should look for a unified account of plural terms.

The treatment of this is complicated by the fact that sometimes predicates can take either singular or plural subjects, creating an ambiguity that must also be accounted for.

(9) Alicia, Betty and Carla lifted the table.

(10) Seven students lifted the table.

These sentences are made true either by multiple individual achievements or by a single lifting by a group of people. If given as an answer to the question "Which students lifted the table?" (9) would be unclear about whether the lifting was collective or distributive.

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If we say 'Schiller and Goethe are poets', we are not really connecting the proper names by 'and', but the sentences 'Schiller is a poet' and 'Goethe is a poet', which have been telescoped into one. It is different with the sentence 'Siemens and Halske have built the first major telegraph network'. Here we don't have a telescoped form of two sentences, but 'Siemens and Halske' designates a compound object about which a statement is made, and the word 'and' is used to help form the sign for this object.

<sup>14</sup> Massey, 1976, p. 103, Dowty 1986, p. 98, Schwarzschild 1996, pp. 14-15, Oliver and Similey 2001, p. 294, and Yi 2002, pp. 25-26 (among others) present similar arguments. (We shouldn't expect this argument to take us too far, though. Some compound NPs are clearly related to compound sentences: *Arnie, Bob or Carlos will stand watch tonight*. It would be a stretch to think that what is possible for 'or' is not possible for 'and'.) This issue brings with it further complications that will be discussed in Chapter 4.

English does not overtly mark distributivity or non-distributivity in (9) or in (10), and we use context, paraphrase or additional clarification to indicate whether the predication is distributive. It may be true that they lifted it together but not true that they lifted it individually (or vice versa), and so it seems that we must mark that distinction in the formal theory we develop, where we want to mark semantically and inferentially significant distinctions.<sup>15</sup> Although a single quantifier can be connected to variables that are in both distributive and non-distributive clauses, we must have a way to mark the difference between distributive and non-distributive interpretations of (10).

The problem becomes even more evident in the quantificational case.

Seven students stepped forward and lifted the table.

The same seven students are to be understood as the subject of 'X stepped forward' and 'X lifted the table' here. (This is not equivalent to "Seven students stepped forward and seven students lifted the table.") So it seems that a single quantifier must be connected to variables that are in both distributive and non-distributive clauses. At the same time, we must distinguish the distributive and non-distributive readings of (10), since it may be true non-distributively and yet false distributively (or vice versa).

The need for marking the distinction is reinforced when we consider other examples. Suppose that Max, Norman and Oscar each weigh 200 pounds. We can say any of these:

- (11) Max, Norman and Oscar weigh 200 pounds.
- (12) Max, Norman and Oscar weigh less than 210 pounds.
- (13) Max, Norman and Oscar weigh 600 pounds.
- (14) Max, Norman and Oscar weigh more than 500 pounds.

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<sup>15</sup> Some have said that sentences like (9) and (10) have an ambiguity in the predicate, for example Dowty 1986, p. 98. But Moltmann (1997), p. 52, point out that we cannot treat this distinction (between distributive and non-distributive readings of a predicate) as an ambiguity in the predicate because predicate ellipsis is readily available. The correct approach, in light of these facts, will be developed in Chapters 2 and 4.

The continued coherence of ordinary numerical predications seems to require that we have a way of indicating that the predication is distributive in (11) and (12) and non-distributive in (13) and (14). Our formal language will incorporate a way of indicating this.

### **Non-distributive relations**

It is important to emphasize that relations can be non-distributively satisfied with respect to one or more argument places.

(15) John Wayne circled wagons 1-9 around the campfire.

(16) John Wayne circled some wagons around the campfire.

The natural way to take this seems to be to regard it as a three-place relation (\_\_\_ circled ... around \*\*\*) that requires plurality in the second place. (Maybe 'requires' is too strong. He could circle a long rope around the campfire.) The first and third places also allow for plurality, if the encircling was a group effort or if several things are in the center. ("John, Buddy and Gabby circled some wagons around the campfire and the food items.")

In representing such relations, I will employ upper-case variables to indicate the possibility of plurality; for example, 'X circled Y around Z', and 'x is one of Y'.<sup>16</sup> With plural quantification, such variables will relate to plural pronouns in English in the way that lower case variables relate to the singular pronouns of English. Thus 'x is one of Y' is very much like 'it is one of them', but we have access to an infinite stock of different pronouns ('x', 'y', 'x<sub>1</sub>', 'x<sub>2</sub>', etc., 'X', 'Y', 'X<sub>1</sub>', 'X<sub>2</sub>', etc.) for making cross-reference relations clear.

All of this indicates that we should really be talking about the plural character of the various places in a relation. For example, when I provided the definition of a distributive predicate, I used a relation that is non-distributive in the second place: 'x is one of Y'.

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<sup>16</sup> Some others (Rayo, Linnebo and Williamson, for example) use *xx* and *yy* (etc.) for the plural variables, instead of upper case letters. That notation seems to have originated in Burgess and Rosen 1997, p. 152. Yi uses 'xs', 'ys', (etc.). There is no significance in this difference of notation. We are all talking about a first-order plural language.

Nevertheless, for reasons of brevity, I will often simply use the phrase "non-distributive predicates," and that should be understood to make reference to predicates and relations that can be non-distributively or non-cumulatively satisfied relative to one or more argument positions.<sup>17</sup>

It will work out best in the long run if we take these "plural" variables to be neutral in number. That is:

$[\exists X: FX] GX$

will express the claim that one or more individuals that are F are G, where F and G are allowed to be non-distributive. For example:

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<sup>17</sup> Strictly speaking, we should now provide a new definition of *distributive*. Here is an informal version of that.

In an n-place relation R, place j is *distributive* iff the meaning of R requires that whenever  $Rt_1 t_2 \dots T \dots t_n$  (with T in the jth place) is true and **a** is one of T, then  $Rt_1 t_2 \dots \mathbf{a} \dots t_n$  (i.e., with a term referring to **a** in the jth place) is true.

Similarly, we can define cumulativity relative to an argument place.

In an n-place relation R, place j is *cumulative* iff the meaning of R requires that whenever  $Rt_1 t_2 \dots T \dots t_n$  (with T in the jth place) is true and  $Rt_1 t_2 \dots T' \dots t_n$  (with T' in the jth place) is true, then if T'' is a term non-distributively referring to the things among T together with the things among T' (and only to those), then  $Rt_1 t_2 \dots T'' \dots t_n$  (with T'' in the jth place) is true.

Here  $t_1, t_2$ , etc. may be any terms, singular or plural, variable or constant, T, T', T'' are plural terms, and **a** is singular. For full generality, we would need to give a definition relative to an assignment to variables. For the definition of cumulativity, we also need a full account of term conjunction. These are elements of the language and semantics to be developed. (In a more formal treatment, we would also clearly differentiate object-language and metalanguage references.)

Some students are surrounding Adams Hall today.

'X are surrounding Adams Hall' is non-distributive.

$[\exists X: SX]$  X are surrounding Adams Hall

In the case in which one student surrounds Adams Hall (in the story "Elasticman Goes to College" perhaps), this sentence will be judged true. If we wish to specify that we are talking about more than one student, we can specifically indicate that we require more than one.

$[\exists X: SX \wedge X \text{ are more than one in number}]$  X are surrounding Adams Hall

In fact, it may be useful to introduce a predicate

$NX$ : X are more than one in number

that we can use whenever we wish to rule out individual satisfiers of a predicate, as the plural in English might be thought to do.

$[\exists X: SX \wedge NX]$  X are surrounding Adams Hall<sup>18</sup>

We will eventually want to consider *among* as a fundamental relation (see Chapters 2 and 6). If we let 'YAX' stand for 'Y are among X', with the singular variant 'yAX' meaning 'y is one of X' (i.e., 'y is among X'), then we can define 'NX' in terms of *among* and *identity*.  $NX =_{df} \exists y \exists z (yAX \wedge zAX \wedge y \neq z)$ .

Multiple quantifications are also possible, and indicate something else about the interaction of quantifiers and plurals:

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<sup>18</sup> I will not try to definitively resolve the question of whether non-distributive plural English sentences have plurality as an implication or at most as an implicature. In the former case, we would include 'NX' routinely as a part of the symbolization of English plurals, and in the latter case we would not. In our formal language we can do it either way, but we have to pick one. In dealing with illustrative examples, I will assume that plurality is an implicature only, and thus not include 'NX' in the formal representation on a routine basis.

(17) Some cowboys circled some wagons around a campfire.

(18) Each cowboy circled some wagons around a campfire.

(18) makes it evident that 'some wagons' is a full-blown quantifier phrase and not a referential expression. If the existential is read as subordinate to the universal in (18), 'some wagons' cannot somehow *refer* to a some particular wagons, because each cowboy's wagons can be distinct from those of the other cowboys.<sup>19</sup> Ultimately we must provide a semantics for plural predicates and plural quantification that allows for such quantifier embedding.

Also, once we allow non-distributive predicates, we will also have the resources for expressing other sentences that involve non-distributive plural predication even though all of the basic predicates in the surface English are distributive. For example, the "non-first-orderizable" Geach-Kaplan sentence:<sup>20</sup>

(21) Some critics admire only one another.

We can represent this in a plural, first-order language in the following ways:

(22)  $[\exists X: X \text{ are critics}] X \text{ admire only one another}$

(23)  $[\exists X: X \text{ are critics}] [\forall y: y \text{ is one of } X] [\forall z: y \text{ admires } z] (z \text{ is one of } X \text{ and } z \neq y)$

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<sup>19</sup> This is why an approach taken in McKay 1994 cannot work. There it is suggested that we can treat each plural quantifier phrase as making reference to some things. But in sentence (13), that is evidently not possible for (the small scope reading of) the quantifier 'some wagons'. (For the same as the reason, a small-scope reading of 'a campfire' cannot be interpreted referentially in that sentence.)

<sup>20</sup> See Boolos 1984, pp. 432-433 (pp. 56-57 in Boolos 1998) for a presentation of a proof, attributed to David Kaplan, that this is not representable in standard (singular) first-order logic.

This is first order because it quantifies with respect to only the basic individuals of the domain (people in this case) and it quantifies only into argument position. There is a non-distributive predicate in each of these representations:

'X admire only one another' in (22)

'z is one of X' in (23)

But the basic predicates that appear explicitly in the English sentence ('X are critics' and 'X admire y') are distributive. We can also represent the Peano induction axiom as a plural, first-order axiom:

$\forall X$  (if 0 is one of X and if  $[\forall y: y$  is one of X] the successor of y is one of X, then

$\forall z, z$  is one of X)

This also contains the non-distributive 'z is one of X'. (See Chapters 2 and 6 for further discussion.)

### **Plurals in philosophy**

In recent years, several philosophers have adopted the language of plurals and non-distributive predicates. For example, George Boolos has argued that some second-order quantification can be understood as plural quantification.<sup>21</sup> David Lewis uses non-

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<sup>21</sup> Boolos 1984, 1985a. Boolos's work has much to tell us about plurals, but it seems that he was not motivated by a general introduction of non-distributive predicates. The predicate 'x is one of Y' is the only primitive non-distributive relation that he employed. He writes 'x is one of Y' as 'Yx', indicating the way in which he assimilates (monadic) second-order quantification to plural quantification involving that one non-distributive relation. But this provides no immediate way to represent the relationship between 'he lifted a piano' and 'they lifted a piano', where the same predicate is applied to an individual and, non-distributively, to some individuals.

distributive plurals and plural quantification in the language that is at the foundation of his reworking of the theory of classes.<sup>22</sup> Peter van Inwagen and others use the language of plurals with non-distributive predicates in posing and answering fundamental questions in

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Boolos's discussion of another example of non-distributive predication, in a discussion of the sentence "The rocks rained down" (1985b, p. 168 in Boolos 1998), is revealing. There he says:

If we have learned anything at all in philosophy, it is that it is almost certainly a waste of time to seek an analysis of "The rocks rained down" that reduces it to a first-order quantification over the rocks in question. It is highly probable that an adequate semantics for sentences like "They rained down" or "the sets possessing a rank exhaust the universe" would have to take as primitive a new sort of predication in which, for example, "rained down" would be predicated not of particular rocks such as this one or that one, but rather of these rocks or those. ... The predication "they M" is probably completely intractable.

I take the last sentence to mean that such predication ("They rained down", for example) is intractable without accommodating non-distributive predication (as first-order predication) in a general way. I hope I am showing how to make that accommodation (without wasting our time).

Alex Oliver's review (Oliver 2000) of Boolos's collected articles is a helpful discussion of these and related issues, especially pp. 871-872. Oliver concludes that "what is needed is a logic of plurals that offers uniform representation of singular and plural forms of a predicate and this is excluded by the hierarchical structure of second-order logic." 'John lifted the table' and 'Some students (together) lifted the table' involve the same predicate, but if plural quantification is taken to be second-order, it is difficult to accommodate that fact.

<sup>22</sup> Lewis 1991. See especially 62-71 for his discussion of the role of plural quantification. See also Rayo and Uzquiano 1999 for another use of plurals in the development of set theory.

metaphysics, such as the question of when some things constitute a single thing.<sup>23</sup> Keith Hossack has developed the language of plurals as a part of a defense of atomism (compositional nihilism).<sup>24</sup>

We cannot use ordinary first-order logical notation in representing what these philosophers say. All point out that plural language is widely used and understood in ordinary discourse. Nevertheless, I think that our exploration of the notation and the semantics for such a language will lead to some interesting insights and surprises. Certainly we should wish to make this as well understood as the more traditional, singularist first-order language, so as to provide a similar resource for sorting out ambiguities and making judgments about the validity of arguments.

Another reason to develop the semantics is to assure ourselves that the semantics of plurals does not itself need to rely on the resources of set theory or anything else that might be a problematic element in the foundation of the philosophical projects that employ such plural language. In Chapter 3 we will formulate the semantics without employing the resources of set theory (and without other singularist assumptions).

We can also make a positive use of our plural language in response to Timothy Williamson's arguments that plural language is not an adequate basis for the expression of propositions that are about everything.<sup>25</sup> A better understanding of the semantics of plurals is required if we are to evaluate his arguments. We will consider those arguments explicitly in Chapter 6 and argue that he has overlooked an important resource for talking about everything.

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<sup>23</sup> *Material Beings*, van Inwagen 1990.

<sup>24</sup> Hossack 2000. Hossack develops the semantics for non-distributive plurals, and there are many points of contact with his paper. There are, however, significant differences in approach.

<sup>25</sup> Williamson 2003, especially 455 - 458. See Chapter 6 for further discussion of this and the related issue of the connection between plurals and second-order logic.